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The Achilles Fatigue Model

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Summary

The objective of a fatigue model is to predict the amount of fatigue damage in a segment of coiled tubing. The Achilles 3.0 fatigue model in Cerberus, developed by Steven Tipton of the University of Tulsa, is based on plasticity theory. It incorporates the geometry and material of the CT and the internal pressure and bending radius, as well as the "state" of the CT before each bending event. Thus, the amount of fatigue damage for a particular bending event depends on the entire bending history of the CT sample.

Fatigue Failure

Fatigue is the tendency of an object, such as coiled tubing, to crack due to repeated bending. Each time a segment of coiled tubing bends or straightens it accumulates more and more fatigue damage. The amount of fatigue for each bending event depends primarily on the material, geometry, bending radius, pressure, and existing fatigue.

When a segment of coiled tubing accumulates too much fatigue, a microscopic crack forms on the inside of the tubing (crack initiation). Eventually, the crack penetrates through the wall of the tubing (fracture). At high pressure, crack initiation and fracture may appear to be simultaneous.

Since the cost of a fatigue failure is very high, it is important to know the fatigue life (bending cycles or trips) remaining in a coiled tubing string before each job. Unfortunately, there is no non-destructive way to measure fatigue. The alternative is to use a fatigue model to predict fatigue life.

It is also important to note that fatigue is a statistical phenomena. In fatigue tests, several identical samples of coiled tubing undergo an identical series of bending events. However, different samples fail after slightly different amounts of bending. In a comparison between test data and fatigue model predictions, the prediction forms a curve through the scattered points of test data.

While a fatigue model can estimate the amount of fatigue damage in a coiled tubing sample, it cannot tell exactly when a specific sample will fail. Any given sample can last longer than predicted, or fail before predicted.

Inputs and Outputs for the Achilles 3.0 Fatigue Model

The Achilles fatigue model in Cerberus, developed by Steven Tipton of the University of Tulsa, is based on plasticity theory. It calculates the fatigue damage produced by a single bending or straightening event. The inputs for the fatigue model fall into three categories:

- basic CT properties—user entered
- the state of the CT, including the current fatigue damage—internally calculated
- bending event properties—user entered

The output of the fatigue model is the state of the CT after each bending event. Note that the state of the CT is both an input and an output for the fatigue model.

For each bending or straightening event, the model takes the basic CT properties, the CT state before the event, and the event properties. The model then calculates the state of the CT after the event, including the cumulative fatigue damage.

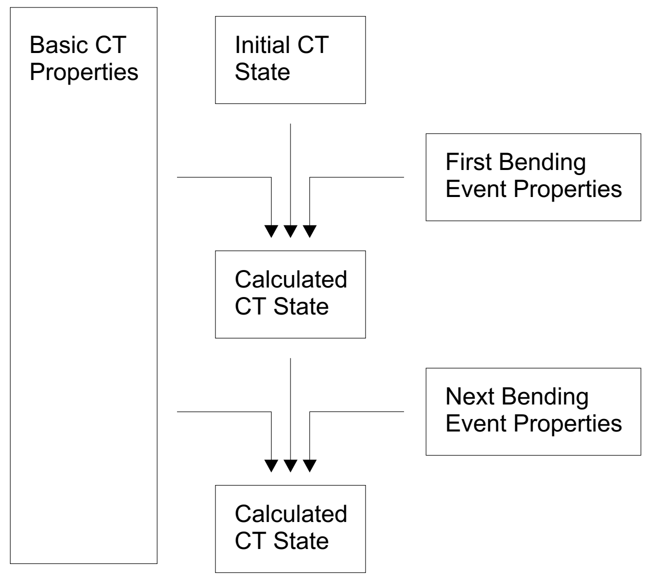

FIGURE 1 Relationship Between Inputs and Outputs

Basic CT Properties

The user enters the basic CT properties. These properties remain constant throughout the life of the CT.

- outside diameter—This input is the original diameter of the tubing. The increased diameter is one of the "state" properties.
- wall thickness—This input is the original wall thickness of the tubing. The decreased wall thickness is one of the "state" properties.
- material type or yield strength—The Achilles fatigue model currently supports 70, 80, 90, and 100 Kpsi yield strength material.

CT State Properties

The fatigue model calculates and stores changes to the CT state properties with each bending and straightening event.

- fatigue damage—This property is the cumulative fatigue damage or "used life" of the CT string. For new CT, the fatigue damage is zero (its used life is 0%).
- grown diameter—This property is the predicted increased tubing diameter. It is calculated from the original diameter and the estimated strain components. For new CT, the grown diameter is the same as the original diameter.
- thinned wall—This property is the calculated reduced wall thickness. It is calculated from the estimated strain components. For new CT, the thinned wall is the same as the original wall thickness.

- stress (axial, radial, and hoop)—These are the orthotropic stress components on the tensile side of the tubing wall. For new CT the stress is zero (0,0,0).
- strain (axial, radial, and hoop)—These are the orthotropic strain components on the tensile side of the tubing wall. For new CT the strain is zero (0,0,0).
- back stress (axial, radial, and hoop)—These are the deviatoric back stress components that define the location of the center of the kinematic yield surface in deviator space, corresponding to the state of stress on the tensile side of the tubing wall. For new CT the back stress is zero (0,0,0).

Bending Event Properties

The user enters the bending event properties for each for each bending or straightening event.

- pressure—This input is the internal pressure at the time the event takes place. It must be a positive value.
- bending radius—This input is the bending radius over which the tubing is bent or from which it is straightened. The model assumes that the tubing is going from completely straight to completely bent, or completely bent to completely straight.

Fatigue Damage Predictions

Cerberus expresses the cumulative fatigue damage as the "used life" of the string. New CT with no fatigue damage has a used life of 0%. Each bending event the CT undergoes increases the used life. When the used life reaches 100%, failure is assumed to occur.

Note, however, that a CT sample can survive past 100% of its used life, or it can fail before reaching 100%. Since samples do fail before reaching a predicted life of 100%, it is important to use an additional safety margin.

A typical point to take remedial action (either retire the string or cut out the area of high fatigue) is when the used life reaches 80%.

Calculations in the Achilles 3.0 Fatigue Model

For each bending or straightening event Achilles performs a series of calculations to compute the fatigue damage, as well as other changes to the CT state.

1. The incremental axial strain, hoop stress, and radial stress are calculated based on the previous CT state, the CT geometry, and the bending event properties.

2. The remaining stress and strain components, as well as the diameter growth and wall thinning, are calculated in a plasticity routine based on a single kinematic von Mises yield surface.

3. The incremental fatigue damage is calculated using the effective strain amplitude. Finally, the incremental fatigue damage is added to the previous fatigue damage.

Axial Strain, Hoop Stress, and Radial Stress

For each bending or straightening event, Achilles first calculates the incremental axial strain, hoop stress, and radial stress.

To calculate the incremental axial strain, Achilles determines which event is taking place (bending or straightening) by checking the axial strain. If ε_x is initially zero, then the tubing is straight and thus bent during this event. The axial stain increment of $\Delta\varepsilon_x$ is computed from the tubing dimensions, D and t, and the radius of curvature, R:

$$\Delta\varepsilon_x = +\frac{(D-t)}{2R}$$

Eq 1

The radius of curvature in this relation is assumed to be that of the neutral bending axis (i.e., the curvature of the centroidal axis of the tube). Eq 1 actually computes the strain at the mid-thickness radial location, based on the assumption that plane sections remain plane. It represents an average of the axial strain through the wall thickness, since the actual strain varies linearly. Eq 1 also assumes that the tubing is going from a completely straight condition, just prior to the event, to full curvature at the end of the event.

If the axial strain is initially non-zero, then a bending strain increment is defined that will return it to zero (completely straight) again. The bending stain increment that does this is

$$\Delta\varepsilon_x = -\varepsilon_x$$

Eq 2

The hoop stress and radial stress are calculated from internal pressure, *P*. Both of these quantities vary nonlinearly through the wall thickness. Therefore, an averaged value is computed using the following equations.

$$\sigma_h = \frac{P\left(\frac{D}{t} - 2\right)}{2}$$

Eq 3

$$\sigma_r = \frac{-P\left(\frac{D}{t} - 2\right)}{2\left(\frac{D}{t} - 1\right)}$$

Eq 4

The values of the hoop and radial stress components given in Eq 3 and Eq 4, respectively, represent updated quantities based on the pressure at the time of the event. Incremental quantities are computed, based on the state of the CT before the bending or straightening event, as shown in Eq 5 and Eq 6.

$$\Delta\sigma_h = \frac{P\left(\frac{D}{t} - 2\right)}{2} - \sigma_h$$

Eq 5

$$\Delta\sigma_r = \frac{-P\left(\frac{D}{t} - 2\right)}{2\left(\frac{D}{t} - 1\right)} - \sigma_r$$

Eq 6

Axial Stress, Hoop Strain, Radial Strain, Diameter Growth, and Wall Thinning

Once the incremental axial strain, hoop stress, and radial stress are known, Achilles computes the remaining stresses and strains and the changed CT geometry. The calculations are based on a single kinematic von Mises yield surface that is prescribed to translate within a fixed von Mises limit surface. Garud's hardening rule[4,5] is used to specify the direction of the yield surface translation, and thus the components of the backstress, following each event. The flow rule used to compute the individual plastic strain components is based on an empirical method. (Standard rules have been shown to overpredict transverse strains associated with circumferential growth and reduced wall thickness.)

The magnitude of the kinematic yield surface for each material type is defined based on a piecewise linear characterization of the cyclic stress-strain curve. When the stress state lies within the yield surface, or attempts to pass through it, stresses and strains are related by isotropic elasticity theory (Hooke's law). The value of stress used to define the yield surface represents the point at which stresses and strains are no longer linearly related.

The cyclic stress-strain curve is defined by fitting a Ramberg-Osgood relation through the data points of cyclically stable stress amplitude versus strain amplitude. These data are generated from low cycle fatigue testing[3]. The stress-strain relation is given by:

$$\varepsilon_a = \frac{\sigma_a}{E'} + \left(\frac{\sigma_a}{K'}\right)^{\frac{1}{n'}}$$

Eq 7

The piecewise linear approximations were determined by inspection and engineering judgement. The piecewise linear curves are defined from their breakpoints, which are the yield point and the strain amplitude corresponding to the limit stress. The slope of the section between the elastic curve and the limit curve are used to define the plastic modulus relating the stress increment to the strain increment in the flow rule.

The flow rule is based on the concept of normality[2] relative to coiled tubing. However, the flow rule used in Achilles is modified in the following manner. First, for each bending and straightening event, the increments of stress and strain are found that correspond to the outward normal to the yield surface at the current stress state. If bending event is occuring, then the normality based strain increments are used to update the state of stress and strain, and no change in the diameter or wall thickness is estimated. (Physically, it is difficult to measure the tube diameter while in the bent condition.) However, the axial strain increment during a straightening event is compressive, causing the stress state to lie on the compressive side of the von Mises yield surface. The exterior normals on this side of yield surface are extremely high, tending to cause an overestimation of positive circumferential strain increment. Therefore, when a straightening event occurs, the strain increments associated with the event are modified such that the growth rate corresponds with empirically based observations of data from constant pressure CT testing. The modification to the yield surface normal is defined to assure that the tubing cross section remains constant, effectively neglecting incremental axial elongation of the tubing. (Recent research has indicated that tubing elongation has a only a secondary effect—less than one percent in the vast majority of field applications—on transverse dimensional changes.) These suggest that the hoop strain growth rate can be related to the tube loading by

$$\frac{\Delta\varepsilon_h}{\Delta N} = 5.072277\left(\frac{\sigma_h}{\sigma_{yld}}\sqrt{\Delta\varepsilon_x}\right)^{2.686058}$$

Eq 8

The hoop strain computed from normality, $\Delta\varepsilon_h'$, is modified to $\Delta\varepsilon_h$ for this cycle, according to Eq 8. The radial strain increment computed from normality, $\Delta\varepsilon_r'$, is thus modified using the following relation

$$\Delta\varepsilon_r = \Delta\varepsilon_r'\left(\frac{\Delta\varepsilon_h}{\Delta\varepsilon_h'}\right)$$

Eq 9

These components are used to compute the grown diameter, *D'*, based on the assumption that the cross sectional area remains equal to its original value[2]. The relation used to make this calculation is

$$D' = \frac{2(D + t\varepsilon_r)}{(2 + \varepsilon_r)}$$

Eq 10

and the thinned wall, *t'*, is computed from the relation

$$t' = t(1 - \varepsilon_r)$$

Eq 11

If an increment of stress attempts to exceed the limit surface, the routine divides into a portion required to just reach the von Mises limit surface. The state of stress and strain are updated to this point. The portion of the stress increment that exceeds the yield surface is used to compute the remaining strain increment based on zero work hardening. The state of stress is returned to the limit surface according to the final value of the hoop and radial stress, which are controlled by the pressure at the time of the event (Eq 3 and Eq 4). Since the state of stress is orthotropic at the tensile side of the tubing (i.e., having only axial, hoop and radial components) then the value of the axial component is exactly specified by the hoop, radial and limit stress. The backstress components are computed for consistency with the final stress state on the limit surface.

Fatigue Damage

Once Achilles calculates all the stress and strain components, it calculates the fatigue damage.

First, the mean and amplitude components of each strain component are computed from Eq 12 and Eq 13, respectively,

$$\varepsilon_{i,m} = \frac{\varepsilon_{i,updated} + \varepsilon_{i,initial}}{2}$$

Eq 12

$$\varepsilon_{i,a} = \frac{\varepsilon_{i,updated} - \varepsilon_{i,initial}}{2}$$

Eq 13

where the subscript, *i*, represents *x*, *h*, or *r*, for the axial, hoop or radial directions, respectively.

Next, an equivalent von Mises mean and amplitude strain component are computed from the individual strain components, according to the equations below.

$$\varepsilon'_m = 0.4714\sqrt{(\varepsilon_{x,m} - \varepsilon_{h,m})^2 + (\varepsilon_{h,m} - \varepsilon_{r,m})^2 + (\varepsilon_{r,m} - \varepsilon_{x,m})^2}$$ **EQ 14**

$$\varepsilon'_a = 0.4714\sqrt{(\varepsilon_{x,a} - \varepsilon_{h,a})^2 + (\varepsilon_{h,a} - \varepsilon_{r,a})^2 + (\varepsilon_{r,a} - \varepsilon_{x,a})^2}$$ **EQ 15**

Then, the equivalent von Mises strain components are used to formulate an **effective strain amplitude**, according to the following relation:

$$\varepsilon_{a,eff} = \varepsilon'_a (1 + \varepsilon'_m)^C$$ **EQ 16**

The effective strain amplitude is defined in order to be directly compatible with the baseline fatigue damage function for the coiled tubing material, namely the low cycle fatigue strain-life curves, as estimated from strain-controlled tests on axial coupons[3]. The exponent, *C*, in Eq 16 is defined as a function of the prevailing hoop stress and the axial strain amplitude for the event, as well as the equivalent von Mises mean strain for the event in the following manner.

$$P = \frac{\sigma_h}{\sigma_{yld}}\sqrt{2\varepsilon_{x,a}}$$
$$if \quad P < P_o \; then : C = C_o + [(C_1 + gP^2 + hP^3) - C_o]e^{(-40\varepsilon'_m)}$$
$$if \quad P \geq P_o \; then : C = C_o + A(P - P_o)^B$$ **EQ 17**

The coiled tubing parameters defining the relations in Eq 17 (P_O, C_O, C_1, *g*, *h*, *A*, and *B*) are determined from analysis of constant pressure fatigue data from coiled tubing samples tested in laboratory fixtures for each material type. Eq 17 represents a three dimensional plot of *C* versus *P* on one axis, and ε'_m on the other.

The effective strain amplitude, defined in Eq 16, is considered to be a “fatigue damage parameter” for coiled tubing. It contains detailed information about not only the severity of the event under consideration but also about the history of the tubing deformation, in the form of the equivalent von Mises mean strain. The effective strain amplitude is used to compute fatigue damage for a complete bending and straightening cycle, N, by using the strain-life relation for the material, considered to be a baseline fatigue damage function, given by:

$$\varepsilon_{a,eff} = \frac{\sigma_f'}{E'}(2N)^b + \varepsilon_f'(2N)^c$$

EQ 18

This relation is referred to as the Manson-Coffin equation. The cyclic modulus, E', is used in order to be consistent with Eq 7. The parameters σ_f' and ε_f' are the fatigue strength and ductility coefficients, respectively, and b and c are the fatigue strength and ductility exponents, respectively. These four parameters vary with coiled tubing material and failure criterion (fracture or crack initiation). The values for fracture were estimated from tests that included the formulation of a crack completely through the cross section of the low cycle fatigue axial coupon samples, whereas the values for crack initation were based on the onset of macroscopically detectable cracking.

Since a bending or straightening event is only one half of a total cycle, as defined in Eq 18, then the amount of life expended by the event is considered to be

$$F_i = \frac{1}{2N}$$

EQ 19

This is equivalent to assuming that a single event expends one half of the damage that would be expended by a total cycle. The damage is updated updated prior to leaving the Achilles by adding F_i to the previous damage. Damage is considered to accumulate until the total quantity, F, reaches 100%.

$$F = \sum F_i = 100\% \rightarrow FAILURE!$$

EQ 20

Defining and summing fatigue damage in this manner is consistent with the concept of cumulative damage as defined by Miner's rule, which is also called "linear" damage summation. This means that under constant operating conditions (e.g., constant pressure and bending strain amplitude, for coiled tubing) an identical quantity of life is expended by each cycle of loading. However, it is apparent by inspection of Eq 16 and Eq 17, that the effective strain amplitude will increase throughout the life of the component, which causes more damage to be computed by cycles later in life, than they caused earlier in life. This is reasonable, in view of the diametral growth and wall thinning that occur in the tubing, as well as the inherent nature of fatigue damage development which includes the nucleation and growth of microscopic cracks.

Life at Low Pressures

The form of the coiled tubing exponent, *C* (Eq 16 and Eq 17), cause life predictions to **increase** with pressure for very low pressures, contradictory to the conventional observation that increased pressure is damaging to coiled tubing. This occurs only up to a moderately low pressure, usually on the order of 500 to 1500 psi, and usually represents a percentage life difference on the order of 10 to15 percent. At higher pressures, life is predicted to decrease as pressure increases, as expected. This tendency was formulated into the model on the basis of trends exhibited by numerous data sets. A physical explanation might be that moderate pressures could offer structural support during bending, resulting in a more uniform distribution of bending strains without activating significant circumferential growth mechanisms. However, significant research would be required to explain the phenomenon with confidence. Finally, notice the exponential term in Eq 17. This term eliminates the low pressure modification just described, if samples are cycled at higher pressures prior to low pressure cycling. The notion that life improvement can be expected with increasing low pressures is based on observation of numerous **constant pressure** data sets. There are no data to validate this notion if low pressure cycling is preceded by cycles at higher pressures. The higher pressures would result in accumulated plastic straining, which would be reflected in the term ε'_m. Future work should investigate variable pressure cycling, which occurs routinely in the field.

Applying the Fatigue Model to a CT String

The fatigue model calculates the fatigue damage for only a single spot on a CT string. However, every spot on a CT string can have different properties and can experience different bending events.

Cerberus obtains a profile of the fatigue damage along the length of a CT string by dividing the CT string into short, manageable segments. For each job, Reel-Trak uses the geometry of the surface equipment and the CT depth to track the position of each segment relative to the bending locations. When a segment bends or straightens, Reel-Trak applies the Achilles fatigue model to the segment.

Bending Events on Land

In almost all CT operations there are six bending events for each segment of CT as it runs in hole and pulls out of the hole.

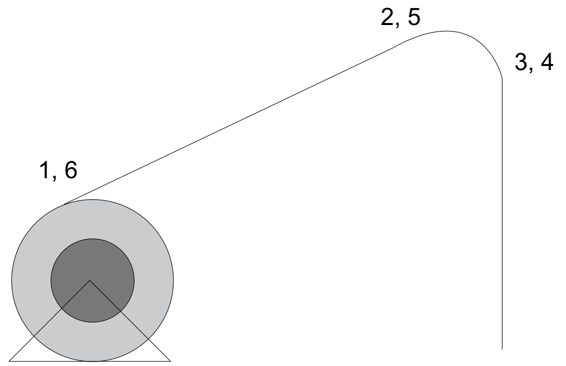

FIGURE 2 Standard Bending Events for CT

1. RIH off the reel - bent to straight
2. RIH onto the gooseneck - straight to bent
3. RIH off the gooseneck onto the injector - bent to straight
4. POOH from the injector onto the gooseneck - straight to bent
5. POOH off the gooseneck towards the reel - bent to straight
6. POOH onto the reel - straight to bent

Note that all of the fatigue damage occurs at the surface, and none occurs in the well. The bending radius in the wellbore is too large to affect fatigue.

Bending Events Offshore

In offshore operations with a wave compensation system, the CT experiences the same bending events as on land, plus additional bending events due to heave. The additional bending events occur when the CT is stationary or moving very slowly. Even though the CT is stationary at the injector, tubing spools on and off the reel to compensate for the movement of the movement of the waves. This movement at the reel can also cause additional bending at the guide arch.

When the CT moves at or above a critical velocity, the movement of the pipe is not influenced by the wave compensation system, and there are no additional bending events.

Unusual Bending Events

In some situations the CT can experience different bending events from the standard ones. Since these situations are so rare, Cerberus does not address them.

- Pipelines and Flowlines—Normally any bending in the well is too gradual to affect fatigue. However, operations in pipelines and flowlines can involve much tighter curves which do affect fatigue.
- Specialized surface equipment—Certain purpose-built CT drilling equipment use unusual surface equipment to decrease the number of bending events or greatly increase the bending radii.

Additional Fatigue Factors

Welds

Welds are weaker than base pipe and tend to fail after fewer cycles. Cerberus addresses welds by assigning each weld a derating factor. The derating factor changes the rate at which fatigue accumulates.

The derating factors in Cerberus come from the 1995 Weld Joint Industry Project. They are based on two main factors which affect the strength of a weld:

- the type of weld (bias weld, manual butt weld, orbital butt weld)
- the wall thicknesses being joined (same or different)

Bias welds are stronger than orbital butt welds, which are stronger than manual butt welds. Welds joining segments with the same wall thickness are stronger than welds joining different wall thicknesses.

Miscellaneous Factors

The Achilles fatigue model does not address several other factors which can influence the fatigue life of a CT string. These factors include corrosion, axial load, rotation, dents, scrapes, and other physical deformities. As these conditions are difficult to quantify and vary wildly from string to string, they have not been researched extensively and are not addressed directly in the fatigue model.

You can address these issues in Cerberus by identifying "zones" along the length of the string. Each zone has a derating factor, which changes the rate at which fatigue accumulates. Note however that choosing a derating factor for a zone is very subjective.

Trends in Fatigue

Fatigue life increases with

- increasing wall thickness
- decreasing CT diameter
- increasing bending radius
- decreasing internal pressure
- increasing material yield strength

You can take advantage of these trends to minimize the effects of fatigue when designing a new CT string and when using an existing one.

- use a thicker wall thickness
- use a smaller outside diameter
- use a yield strength appropriate for the anticipated pressures
- place the CT on the largest diameter reel, or use a reel core expander
- use welds that join the same wall thicknesses
- use a larger guide arch
- reduce the internal pressure during trips
- minimize cycling the same segment over the guide arch
- avoid cycling segments that currently have high fatigue

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